II. THEORY

We first pose the problem of shock temperature and formulate a theoretical basis for its solution. Let e, s, U, and u denote specific energy, specific entropy, shock velocity, and particle velocity, and let subscript o denote the constant state of stationary fluid in front of the shock. Then the Rankine-Hugoniot jump[6] conditions relating shocked and unshocked states,

$$vU = v_o(U - u) \tag{1}$$

$$uU = v_o(p - p_o) \tag{2}$$

$$puv_o = U(e - e_o + \tfrac{1}{2}u^2) \tag{3}$$

express the balance of mass, momentum, and energy across the shock discontinuity, and the inequality

$$s(e,v) > s(e_o,v_o)$$

expresses the second law of thermodynamics for the irreversible shock process.

Eliminating U and u from Eq. 3 gives the Hugoniot equation[7]

$$e - e_o = \tfrac{1}{2}(p + p_o)(v_o - v). \tag{4}$$

If an (e-p-v) equation of state satisfies the condition $(\partial^2 p/\partial v^2)_s > 0$, then Eq. 4 with $v < v_o$ defines the locus of compressed states on the (e-p-v) surface that can be reached from an initial condition (e_o, p_o, v_o) by single shocks. The (e-p-v) equation of state and Eq. 4 define this locus of shocked states as a curve in the (p,v) plane, $p = p_H(p_o, v_o, v)$, which passes through the point (p_o, v_o) and is called the Hugoniot curve centered at (p_o, v_o). The elimination of u from Eqs. 1 and 2 gives the equation of the Rayleigh line,

$$p - p_o = (U/v_o)^2 (v_o - v). \tag{5}$$

Since a shocked state satisfies Eqs. 4 and 5, the intersection of the Hugoniot curve centered on (p_o, v_o) and the Rayleigh line of slope $-(U/v_o)^2$ passing through (p_o, v_o) defines the mechanical thermodynamic state (p,v) behind a shock propagating at constant velocity U into a stationary state (p_o, v_o).